ANALYTICAL AND EXPERIMENTAL STUDIES OF RADIATIVE HEAT EXCHANGE IN THE MUFFLE CHAMBERS OF MULTICHANNEL FURNACES

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An analysis is given for the process of radiative heat exchange between the element of a muffle furnace and the fired material. On the basis of the analysis and experimental data we have derived a system of equations which uniquely define the conditions of heat transfer within the furnace.

In the heat treatment of thin ceramics we note a trend toward the use of multichannel furnaces with the fired materials placed in a single row, in preference over the use of muffle–tunnel furnaces. Efficient utilization of multichannel furnaces makes it necessary to know the conditions of heat transfer which prevail in the muffle chamber of the furnace.

The purpose of this investigation is the study of the transfer of heat between the muffle chamber and the material to develop methods of calculating the optimum heating rate, proceeding from the physico-mechanical properties of the fired material and the structural features of multichannel furnaces.

The muffle chambers of multichannel furnaces are rectangular elongated tunnels lined with shaped refractories forming a number of narrow parallel channels (slots) through the height of the tunnel, with the material sliding along the bottoms of these channels. The muffle chamber is broken down lengthwise into zones of heating, firing, and cooling.

Two external side surfaces are used to transmit heat to the muffle chamber and this heat is propagated by conduction and radiation. The transfer of heat from the muffle chamber to the material is accomplished by radiation about the inside perimeter of the muffle-chamber element.

Figure 1 shows the diagram for the muffle-chamber element for a single channel.

Heat is radiated to the surface of the material from the two side walls on the inside and from the cover on the top, which is a double extended surface whose base is formed by the side walls of the channel.

We make the following assumptions in this analysis: 1) the process of heat transfer from the channel perimeter to the material is accomplished by radiation exclusively; 2) the emissivities of the muffle-chamber materials and of the fired materials are assumed equal to unity; 3) the heat flow within the fired material in the x-direction is negligibly small; 4) the channel is divided lengthwise into individual segments. Within the limits of each segment the temperature T_2 of the side surface of the muffle chamber is a constant, i.e., the smooth variation in temperature over the length of the channel changes into a stepwise variation, making it possible to change from a three-dimensional to a two-dimensional heat-transfer scheme.

The starting point of our analysis is the law of the conservation of energy, which in this case assumes the form of a system of balance equations for the transfer of heat by pure conduction within the volume of the muffle-chamber element and by radiation within the space of the channel bounded by the inside perimeter.

On the surface of the fired material we will isolate an area dx which receives heat exclusively as a consequence of radiation from the bases and the extended surface. Heat is spent on the natural radiation and on the internal heat requirement q_x.

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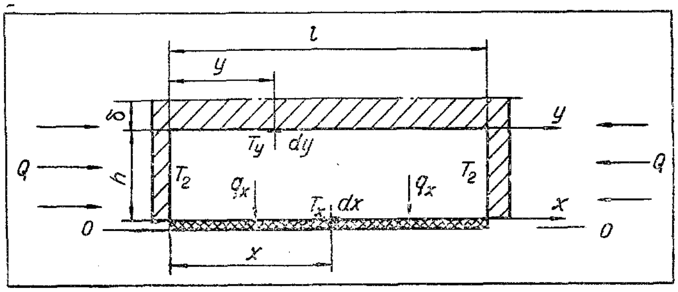

Fig. 1. Diagram of the transfer of heat in the muffle-chamber element.

Let us compile the first balance equation of the system

$$\sigma_0 T_x^4 dx + q_x dx = h\sigma_0 T_2^4 \varphi_{1,dx} + h\sigma_0 T_2^4 \varphi_{2,dx} + \sigma_0 \int_0^l T_y^4 dy \varphi_{dy,dx}, \tag{1}$$

where $\varphi_{1,dx}$, $\varphi_{2,dx}$, and $\varphi_{dy,dx}$ are angle factors.

According to the principle of angle-factor reversibility

$$h\varphi_{1,dx} = dx\varphi_{dx,1},$$
$$h\varphi_{2,dx} = dx\varphi_{dx,2},$$
$$dy\varphi_{dy,dx} = dx\varphi_{dx,dy}.$$

We find the values of the angle factors $\varphi_{dx,dy}$, $\varphi_{dx,1}$, and $\varphi_{dx,2}$ from the formulas proposed by Jakob [1] to determine the angle factors between materials of infinite length in a direction perpendicular to the plane of the drawing:

$$\varphi_{dx,dy} = \frac{1}{2} \frac{h^2}{[h^2 + (y - x)^2]^{3/2}} dy;$$

$$\varphi_{dx,1} = \frac{1}{2}\left(1 - \frac{l - x}{\sqrt{h^2 + (l - x)^2}}\right);$$

$$\varphi_{dx,2} = \frac{1}{2}\left(1 - \frac{x}{\sqrt{h^2 + x^2}}\right).$$

We will substitute the values of the angle factors into (1) and we will reduce terms of the equation by dx:

$$\sigma_0 T_x^4 + q_x = \frac{\sigma_0 T_2^4}{2}\left(2 - \frac{l - x}{\sqrt{h^2 + (l - x)^2}} - \frac{x}{\sqrt{h^2 + x^2}}\right) + \frac{\sigma_0}{2}\int_0^l T_y^4 \frac{h^2}{[h^2 + (x - y)^2]^{3/2}} dy. \tag{2}$$

To compile the second equation, let us examine the conditions of heat exchange between the double extended surface and the ambient medium.

On the plane of the extended surface we will isolate an area dy, situated at a distance y from the channel wall. The heat reaches dy as a consequence of radiation from the first and second bases and from the fired material. In addition, heat is supplied to the area dy as a consequence of heat conduction through the extended surface. Heat is expended on natural radiation. Using the basic principle of conservation of energy and evaluating the term characterizing the thermal conductivity, using the Fourier law for an extended surface of unit width, we obtain in the usual fashion

$$\sigma_0 T_y^4 dy = \lambda\delta \frac{d^2 T_y}{dy^2} + \sigma_0 T_2^4 h\varphi_{1,dy} + \sigma_0 T_2^4 h\varphi_{2,dy} + \sigma_0 \int_0^l T_x^4 dx \varphi_{dx,dy}. \tag{3}$$

Applying the principle of angle-factor reversibility and substituting the values of these factors into (3), we obtain the following expression for the second balance equation:

$$\sigma_0 T_y^4 = \lambda\delta \frac{d^2 T_y}{dy^2} + \frac{\sigma_0 T_2^4}{2}\left(2 - \frac{l - y}{\sqrt{h^2 + (l - y)^2}} - \frac{y}{\sqrt{h^2 + y^2}}\right) + \frac{\sigma_0}{2}\int_0^l T_x^4 \frac{h^2}{[h^2 + (x - y)^2]^{3/2}} dx. \tag{4}$$

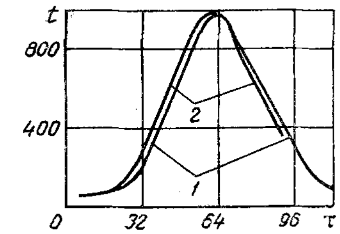

Fig. 2. Temperature curve for the firing of a ceramic plate in a multichannel furnace (t, °C, τ, min): 1) at the center of the fired material; 2) at the edges of the fired material.

The system of equations (2) and (4) can be brought to a convenient dimensionless form through introduction of new variables, i.e.,

$$\Theta = \frac{T}{T_2};\quad X = \frac{x}{l};\quad Y = \frac{y}{l};\quad \bar{q}_x = \frac{q_x}{\sigma_0 T_2^4}.$$

In dimensionless form the system is written as follows:

$$\Theta_x^4 + \bar{q}_x = 1 - \frac{1-X}{2\sqrt{\left(\frac{h}{l}\right)^2 + (1-X)^2}} - \frac{X}{2\sqrt{\left(\frac{h}{l}\right)^2 + X^2}} + \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{\Theta_y^4}{\left[\left(\frac{h}{l}\right)^2 + (Y-X)^2\right]^{3/2}}\,dy; \tag{5}$$

$$\frac{d^2\Theta_y}{dy^2} = N_c\left[\Theta_y^4 - \left(1 - \frac{1-Y}{2\sqrt{\left(\frac{h}{l}\right)^2 + (1-Y)^2}} - \frac{Y}{2\sqrt{\left(\frac{h}{l}\right)^2 + Y^2}}\right)\right] - \frac{\left(\frac{h}{l}\right)^2}{2}\int_0^1 \frac{\Theta_x^4}{\left[\left(\frac{h}{l}\right)^2 + (X-Y)^2\right]^{3/2}}\,dX, \tag{6}$$

where h/l is the geometric parameter which is a measure of the height of the base as a ratio of the length of the extended surface; $N_c = \sigma_0 T_2^3 l^2/\lambda\delta$ is a dimensionless parameter of thermal conductivity, representing the ratio of the thermal radiation of a blackened extended surface at a temperature T_2 to its normal thermal-conductivity component.

We determine the boundary conditions from the symmetry conditions of the system:

$$[\Theta_y]_{y=0} = [\Theta_y]_{y=1};\quad \left[\frac{d\Theta_y}{dy}\right]_{y=\frac{1}{2}} = 0.$$

The system of two equations contains three unknowns: Θ_y, Θ_x, and $\bar{q}_x$.

To find a uniquely defined solution we must present $\bar{q}_x$ in the form of a function of other variables or it must be given a constant value.

The nature of the heat distribution over the length of the channel in a multichannel furnace, established over the width of the fired material (Fig. 2), shows that in the basic heating and cooling segments for the fired material the slope of the temperature curves is identical, indicating a stable requirement of heat through the width of the fired material, i.e., we are dealing with a regular thermal regime of the second kind:

$$\bar{q}_x = \bar{q}_c = \text{const.}$$

However, in this case the numerical solution of the system of integrodifferential equations is made difficult, since we do not know the nature of the functions Θ_y and Θ_x.

We know only of the special case of a solution for $T_x = 0$°K, in which case the system is markedly simplified and assumes the form of a second-order differential equation

$$\frac{d^2\Theta_y}{dy^2} = N_c\left[\Theta_y^4 - (\varphi_{y,1} + \varphi_{y,2})\right],$$

whose numerical solutions were derived on an electronic digital computer by Sparrow and Eckert for a black surface [2], and by Zhulev and Kosarenkov for a gray surface for which $\varepsilon = 0.7$ [3].

To determine the nature of the distribution for Θ_y and Θ_x we undertook certain tests on an experimental installation, and these enabled us to determine the nature of the curves for Θ_y and Θ_x, in addition to helping us in the solution of the system of integrodifferential equations.

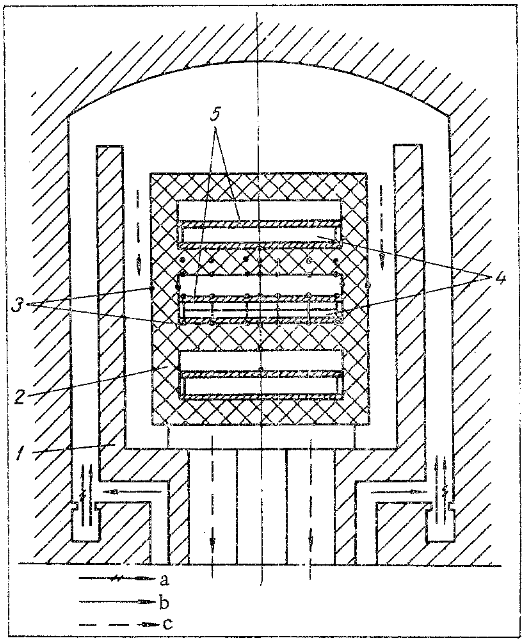

Fig. 3. Diagram of the experimental installation: 1) furnace; 2) muffle chamber; 3) thermocouples; 4) calorimeters; 5) heat insulation for the calorimeters; a) natural gas; b) air; c) flue gases.

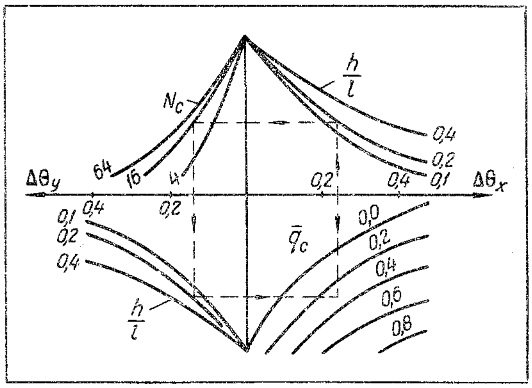

Fig. 4. Nomogram.

The experimental installation consisted of a periodic-heating furnace, a model of the muffle chamber lined with carborundum shaped refractories, calorimeters, heat insulation for the calorimeters, a system of water supply and removal, and measuring equipment. A diagram of the installation is shown in Fig. 3.

Chromel–Copel thermocouples were positioned about the entire channel perimeter and along the calorimeter heat-insulation surface. The side surfaces on the outside were measured with platinum–platinorhodium thermocouples.

The calorimeters were installed so as to make it possible to determine the quantities of heat absorbed by some segment of the heat-receiving surface, as well as to stabilize the heat-transfer process in time. The temperature difference between the influx and drainage water was determined by means of differential Chromel–Copel thermocouples.

The experimental installation was designed so as to provide for lateral heat influx. The substantial length of the model (700 mm) and the good insulation of the ends of the muffle chamber reduced the end boundary effects to the minimum. The tests were performed as soon as a steady-state temperature and heat-flow distribution was achieved in the muffle chamber, with the temperature of the outside side surface stable. The tests differed from each other in the temperature specified for the side surface (the measurements were performed for a temperature range of 600-1200°C for the side surface on the outside), as well as in the form of the heat insulation. The experiments were performed with two configurations of the lateral cross section of the heat insulation, with a geometric parameter $h/l = 0.1$, and with the thermal conductivity varying in the range $N_c = 15$-80.

In the first stage of the investigation we used phosphatoceramoperlite plates $S = 8$ mm ($\lambda = 0.087 + 15 \cdot 10^{-5}$ t W/m·deg) as the heat insulation, which simulated the initial stage of the heating, corresponding to the conditions

$$\bar{q}_x = \frac{T_x - T_0}{R\sigma_0 T_2^4}.$$

On the basis of the data for the distribution of the heat flows, which we derived in the first stage of the investigation, we chose a heat-insulation profile which provided for a constant heat flow through the width of the insulation, i.e.,

$$\bar{q}_x = \bar{q}_c = \text{const}.$$

The measurement results for the temperatures and heat flows, as well as the relative values for $\bar{q}_c$ = const, grouped for the separate horizontal planes setting the temperature distributions through the width of the extended surface and the heat installation, are given in Tables 1-3.

TABLE 1. Temperature Distribution through the Width of the Extended Surface

Temperature, °C	Distance from the measurement point to the base, Y							
	0,054	0,234	0,414	0,505	0,597	0,777	0,946	1,000
T, °K	849	753	698	690	700	758	856	863
Θ	0,985	0,873	0,808	0,800	0,810	0,878	0,992	1,00
T, °K	1057	926	845	839	853	933	1072	1083
Θ	0,976	0,855	0,780	0,775	0,83	0,862	0,990	1,000
T, °K	1224	1059	967	957	972	1066	1236	1247
Θ	0,982	0,850	0,776	0,768	0,780	0,855	0,990	1,000

TABLE 2. Temperature Distribution through the Width of the Heat Insulation

Temperature, °C	Distance from the measurement point to the base, X							
	0,066	0,233	0,414	0,500	0,692	0,767	0,950	1,000
T, °K	804	706	643	635	645	705	819	863
Θ	0,932	0,818	0,745	0,736	0,747	0,817	0,950	1,000
T, °K	1010	890	808	783	794	916	1034	1083
Θ	0,932	0,820	0,746	0,723	0,733	0,847	0,956	1,000
T, °K	1173	1032	938	915	928	1053	1187	1247
Θ	0,940	0,830	0,752	0,735	0,745	0,846	0,952	1,000

TABLE 3. Calorimeter Heat-Flow Values Q, W/Calorimeter

T_2, °K	Calorimeter number					$\overline{q}_c$
	1	2	3	4	5	
863	194	198	192	196	198	0,202
1083	253	253	266	251	261	0,108
1247	345	339	356	353	355	0,084

On the basis of experimental data, using the method of approximate harmonic analysis [4], we derived the relationship describing the nature of the distribution for the relative temperatures through the width of the extended surface and the heat insulation:

$$\Theta_y = 1 - \Delta\Theta_y \sin \pi y,$$

$\Delta\Theta_y$ is the difference between the relative temperatures through the extended surface, between its base and axis of symmetry;

$$\Theta_x = 1 - \Delta\Theta_x \sin \pi x,$$

$\Delta\Theta_x$ is the maximum difference in the relative temperature through the width of the heat insulation.

In the heat treatment of ceramic materials the value of $\Delta\Theta_x$ is a criterion which determines their integrity and straightness; its values are known for specific temperature intervals on the heat-treatment curve [5].

We introduce the relationships derived for Θ_y and Θ_x into the system of integrodifferential equations for coordinate values of Y = 0.5 and X = 0.5 (the relationships for Θ_y and Θ_x are valid for all values of the coordinates X and Y from zero to one, but this substitution simplifies the system), and as a result we derive a system of nonlinear algebraic equations of the following form:

$$(1-\Delta\Theta_y)^4 - \frac{\pi^2 \Delta\Theta_Y}{N_c} = 1 - \frac{0.5}{\sqrt{\left(\frac{h}{l}\right)^2 + 0.5^2}} + \frac{\left(\frac{h}{l}\right)^2}{2} \int_0^1 \frac{(1-\Delta\Theta_x \sin \pi X)^4}{\left[\left(\frac{h}{l}\right)^2 + (X-0.5)^2\right]^{3/2}} dX,$$

$$\bar{q}_c = 1 - \frac{0.5}{\sqrt{\left(\frac{h}{l}\right)^2 + 0.5^2}} + \frac{\left(\frac{h}{l}\right)^2}{2} \int_0^1 \frac{(1 - \Delta\Theta_y \sin \pi Y)^4}{\left[\left(\frac{h}{l}\right)^2 + (Y - 0.5)^2\right]^{3/2}} dy - (1 - \Delta\Theta_x)^4,$$

whose solution is shown by the nomogram in Fig. 4.

Determining the values of $\bar{q}_c$ from the nomogram, we find the optimum heating rate

$$\vartheta = \frac{\sigma_0 T_2^4 \bar{q}_c}{mc}.$$

NOTATION

T	is the absolute temperature;
Θ	is the dimensionless temperature;
$\Delta\Theta$	is the temperature difference through the width of the surface;
q_x	is the power of the internal heat consumption;
$\bar{q}_x$	is the relative power of the internal heat consumption;
ϑ	is the heating rate;
h	is the base height;
δ	is the thickness of the extended surface;
l	is the length of the extended surface;
x	is the coordinate along the surface of the material;
y	is the coordinate along the extended surface;
X, Y	are dimensionless coordinates;
φ	is the angle factor;
R	is the heat resistance of the material;
σ_0	is the radiation constant for a perfect black body;
m	is the mass of the material;
c	is the heat capacity of the material.

Symbols

1, 2	denote the numbers of the base surfaces;
x	is the surface of the material;
y	denotes the extended surface;
0	denotes the midplane of the material.

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